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# Engineering Notes

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## Reduction of Propagated Satellite Yaw Errors Using Orbital Rate Coupling

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### Introduction

MODERATE-ACCURACY attitude determination systems for Earth-pointing satellites typically use Earth and sun sensors for attitude sensing. Since the satellites are Earth-pointing, an Earth sensor can provide continuous roll and pitch attitude data for updating the onboard real-time attitude estimated by a Kalman filter driven by gyro data. Yaw data are provided by a sun sensor, however, due to satellite eclipses and limited fields-of-view, these data are available for only part of each orbit. When yaw data are not available, the yaw estimate must be propagated using gyro data and the onboard Kalman filter.

The general form of the dynamics model for a zero-momentum satellite[1–4] includes the roll/yaw coupling that arises as the satellite pitches over at orbital rate, whereas the attitude control system maintains the Earth-pointing attitude around the orbit. This report shows analytically, via the method described in Ref. 5, how propagated yaw estimation errors can be reduced by using continuous roll attitude measurements in conjunction with orbital rate coupling in a Kalman filter formulation. In essence, the principle of yaw gyrocompassing is embodied in a Kalman filter driven by continuous roll attitude and roll/yaw rate measurements. In the formulation described herein, the structure of the filter is essentially unchanged with and without yaw attitude updates. It is shown that significant reductions in the propagated yaw estimation errors can be achieved during extended periods when no yaw measurements are available.

### System Model

In this development, the body-fixed representation of the state vector and covariance matrix is used,[1] in which the attitude errors are defined as the incremental angles that must be added to the estimated attitude angles to obtain the true angles. Consistent with this definition, the remaining elements of the state vector are the incremental corrections to the gyro drift–bias estimates. Thus, the state vector $\boldsymbol{x}$ is written as $\boldsymbol{x} = (\Delta\varphi, \Delta\theta, \Delta\psi, \Delta b_x, \Delta b_y, \Delta b_z)^T$, where $\Delta\varphi$, $\Delta\theta$, and $\Delta\psi$ are the roll, pitch, and yaw attitude errors, $\Delta b_x$, $\Delta b_y$, and $\Delta b_z$ are the roll, pitch, and yaw gyro drift-bias errors, and the superscript $T$ denotes the transpose operator. The roll (X) axis is generally aligned with the velocity vector, the pitch (Y) axis is generally aligned with the negative orbit normal, and the yaw (Z) axis is generally aligned with nadir. The continuous system dynamics are represented as $\dot{\boldsymbol{x}}(t) = F\boldsymbol{x}(t) + \boldsymbol{w}(t)$, where $F$ is the dynamics matrix and $\boldsymbol{w}(t)$ is a white noise process. The measurement equation is $z(t) = H\boldsymbol{x}(t) + \boldsymbol{v}(t)$, where $H$ is the measurement matrix and $\boldsymbol{v}(t)$ is a white noise process on the measurements. The matrix $F$ describes the kinematic relationship between the states and their rates of change:

$$F = \left[\begin{array}{c|c} \omega^x & -I_3 \\ \hline 0_3 & 0_3 \end{array}\right] \tag{1}$$

Here $I_3$ is the $3\times3$ identity matrix, $0_3$ is the $3\times3$ zero matrix, and $\omega^x$ is the gyro-measured body rate cross product matrix. For a nonslewing Earth-pointing satellite in a circular or near circular orbit, $\omega_x \approx 0$, $\omega_z \approx 0$, and $\omega_y \approx -\omega_0$, where $\omega_0$ is the (constant) orbital rate. This assumes that gyro bias and noise rates are much smaller than orbital rate. With these approximations, $\omega^x$ becomes:

$$\omega^x = \begin{bmatrix} 0 & 0 & \omega_0 \\ 0 & 0 & 0 \\ -\omega_0 & 0 & 0 \end{bmatrix} \tag{2}$$

Roll and yaw are thus coupled through orbital rate. Equations (1) and (2) also show that for nonslewing operation, the dynamics matrix $F$ is approximately constant. The filter implementation can be simplified by neglecting orbital rate, thereby making the $\omega^x$ matrix zero and leading to a very simple state transition matrix.[3] This is done if full three-axis measurements are always available and the orbital rate effect is small, or it is treated as a known pitch gyro bias and subtracted from the pitch gyro readings prior to input to the Kalman filter. However, in the latter case, the orbital rate coupling cannot be used to advantage during periods when yaw measurements are not available.

The measurement matrix $H$ can be expressed as $H_3 = [I_3 \mid 0_3]$. The implicit assumption behind this simple form is that the Earth and sun sensors provide direct measurements of the roll, pitch, and yaw attitude errors independent of the current estimated state. This is not true of the yaw measurement, which must be derived from the location of the sun vector in the sensor field-of-view, the alignment of the sensor on the satellite, the estimated satellite attitude, and orbit knowledge to convert the sun (inertial) attitude measurement to the rotating Earth-frame based attitude. This results in a yaw measurement that is a function of the current estimated state, and a nonlinear estimation problem. For the sake of simplicity, it will be assumed that the system can be linearized such that the measurement matrix $H$ is constant and takes the simple form given here. This allows the use of the conventional Kalman filter formulation, and does not detract from the generality of the results. When the sun is not in the sensor field-of-view, only roll and pitch measurements are available, and the $H$ matrix reduces to rank two, $H_2 = [I_2 \mid 0_2 \mid 0_2]$ where $I_2$ and $0_2$ are the $2\times2$ identity and zero matrices.

The white noise process $\boldsymbol{w}(t)$ represents gyro noise with the following spectral density characterization[2,5]:

$$Q = \left[\begin{array}{c|c} \left(\sigma_w^2 \Delta T + \sigma_v^2\right) I_3 & 0_3 \\ \hline 0_3 & \sigma_u^2 I_3 \end{array}\right] \tag{3}$$

In Eq. (3), $\sigma_w^2$ is the gyro angle white noise variance, $\sigma_v^2$ is the gyro angle random walk (drift rate noise) variance, $\sigma_u^2$ is the gyro rate random walk (drift rate ramp noise) variance, and $\Delta T$ is the Kalman filter update interval for the discrete implementation. Similarly, the

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white noise process $\boldsymbol{v}(t)$ represents measurement noise with the following spectral density characterization[2,5]:

$$R_3 = \begin{bmatrix} \sigma_e^2 \Delta T & 0 & 0 \\ 0 & \sigma_e^2 \Delta T & 0 \\ 0 & 0 & \sigma_s^2 \Delta T \end{bmatrix} \tag{4}$$

In Eq. (4) $\sigma_e^2$ is the Earth sensor roll/pitch measurement noise variance, and $\sigma_s^2$ is the sun sensor yaw measurement noise variance. In general, the roll and pitch noise variances are not equal, but are assumed to be for simplicity. When no sun angle measurements are available, the spectral density matrix becomes $R_2 = (\sigma_e^2 \Delta T) I_2$.

Because the system being discussed is time-invariant, the steady-state covariance matrix can be computed analytically by solving the nonlinear matrix Riccati equation. The solution is obtained using a transformation resulting in a system of linear differential equations[5] that include the matrices $F$, $H$, $R$, and $Q$ defined here. The closed-form solution for the covariance matrix is[5]:

$$P(t) = [\Phi_{21}(t) + \Phi_{22}(t)P(t_0)][\Phi_{11}(t) + \Phi_{12}(t)P(t_0)]^{-1} \tag{5}$$

In Eq. (5), $\Phi_{11}(t)$, $\Phi_{12}(t)$, $\Phi_{21}(t)$, and $\Phi_{22}(t)$ are the $3 \times 3$ submatrices comprising the $6 \times 6$ state transition matrix of the transformed system. This state transition matrix is the solution of a matrix exponential in the matrices $F$, $H$, $R$, and $Q$. $P(t_0)$ is the initial estimate of the covariance matrix.

Two steady-state covariance matrices are computed using this method—one for the case in which full three-axis attitude measurements are available, and the other in which no yaw measurements are available. The matrices $F$ and $Q$ are the same in both calculations, but the matrices $H$ and $R$ are different depending on the available attitude measurements. Because the (3, 3) element of the $6 \times 6$ covariance matrix is the variance of the yaw estimation error, this method can be used to predict the uncertainty in the propagated yaw estimate without performing a Kalman filter simulation. By modeling orbital rate coupling in the dynamics matrix $F$, the effect of this coupling on the yaw estimate can be predicted analytically, as shown by the following example.

## Numerical Example

Table 1 lists the parameters for a moderate-accuracy attitude determination system of an Earth-pointing satellite in a low-Earth orbit utilizing an Earth sensor, a sun sensor, and gyros. This example uses a single-axis sun sensor that provides only yaw information, and with an accuracy the same as that of the Earth sensor. The 67-min yaw nonsensing time $t_p$ ($\frac{2}{3}$ of the orbital period) includes the time spent in eclipse and the outage time because of the limited field-of-view of the sun sensor.

The first step is to compute the steady-state covariance matrix with three-axis measurements available, i.e., prior to the beginning of the yaw nonsensing period. Equation (5) is evaluated at $t = 33$ min, or the orbit period minus $t_p$. The rms estimation errors are the square roots of the diagonal elements of the covariance matrix at this time, computed as $\tilde{\boldsymbol{x}} = (32.6, 32.6, 32.6, 0.072, 0.072, 0.072)^T$, where the first three elements are the rms attitude errors in units of arcseconds, and the next three elements are the rms gyro bias errors in units of arcseconds/second.

Next, the covariance of the two-measurement system at $t = t_p$ is computed using the steady-state covariance matrix from the previous calculation as the initial value of the covariance $P(t_0)$ in Eq. (5).

**Table 1 Attitude determination parameters**

| Parameter | Symbol | Value | Units[a] |
|---|---|---|---|
| Orbit rate | $\omega_0$ | 0.00105 | rad/s |
| Gyro angle white noise variance | $\sigma_w^2$ | $(5.0)^2$ | arcs$^2$ |
| Gyro angle random walk variance | $\sigma_v^2$ | $(2.0)^2$ | arcs$^2$/s |
| Gyro rate random walk variance | $\sigma_u^2$ | $(0.002)^2$ | arcs$^2$/s$^3$ |
| Kalman filter update interval | $\Delta T$ | 10.0 | s |
| Earth sensor noise variance | $\sigma_e^2$ | $(120.0)^2$ | arcs$^2$ |
| Sun sensor noise variance | $\sigma_s^2$ | $(120.0)^2$ | arcs$^2$ |
| Yaw propagation (nonsensing) time | $t_p$ | 67.0 | min |

[a] arcs = arcseconds

In this calculation, the matrices $H_2$ and $R_2$ replace $H_3$ and $R_3$. The computed rms estimation errors are $\tilde{\boldsymbol{x}} = (34.6, 32.8, 171.1, 0.133, 0.075, 0.097)^T$. As expected, the pitch–attitude and pitch–rate errors remain practically unchanged throughout the yaw nonsensing period. Since roll and yaw are coupled, the yaw attitude propagation affects the roll gyro bias estimate directly. Also, since there are no yaw updates, the yaw-gyro-bias-estimation error increases, and this, through the coupling, causes a small degradation in the roll–attitude estimate, even though roll is updated with Earth sensor data throughout. The degradation, however, is more than offset by the reduced yaw-attitude-estimation error. This is illustrated by performing the same calculation with orbital rate set to zero in the dynamics matrix, i.e., no orbital rate coupling. For the three-measurement system, in which continuous yaw updates occur, the steady-state rms estimation errors are unchanged from the previous calculation, showing that orbital rate coupling has no effect on the Kalman filter performance when three-axis measurements are continuously available. However, at the end of the 67-min yaw-nonsensing period, the rms estimation errors are $\tilde{\boldsymbol{x}} = (32.8, 32.8, 449.6, 0.075, 0.075, 0.146)^T$. Thus, without orbital-rate coupling included in the dynamics matrix, the rms yaw-attitude-estimation error at the end of the nonsensing period is 2.6 times greater (449.6 vs 171.1 arcseconds) than the case in which it is included.

## Simulation

A simplified Kalman filter simulation that computes the propagated and updated covariance at each filter update interval was used to validate the analytical results. The discrete Kalman filter equations[5] were evaluated at each filter update time using the parameter values in Table 1 and the discrete versions of the matrices $H$, $R$, $\Phi$, and $Q$. Because the system is time-invariant, a closed-form solution for the discrete transition matrix $\Phi_k$ can be obtained.[4] The simulation spans approximately 2.3 orbits. As in the numerical example, yaw updates from the sun sensor occur only during the first one-third of each orbit, and a propagated yaw estimate is used for the remaining two-thirds of the orbit. The solid-line plots in Fig. 1 show the results of this simulation. Plotted in this figure is the rms estimation error of each element of the state vector, with the three attitude states on the left and the gyro bias estimates on the right. As shown, the yaw error is 33 arcseconds when sun sensor data are available, and reaches approximately 170 arcseconds during the nonsensing part of the orbit. The dashed lines on the same plots show the effect of removing orbital rate from the dynamics modeling. Roll and yaw are no longer coupled, and the yaw estimation error during the nonsensing period reaches approximately 460 arcseconds. Without the roll coupling, roll attitude updates from the Earth sensor have no effect on the yaw estimate, and the propagation is more sensitive to the gyro errors. The results shown in Fig. 1 confirm the analytical results presented previously.

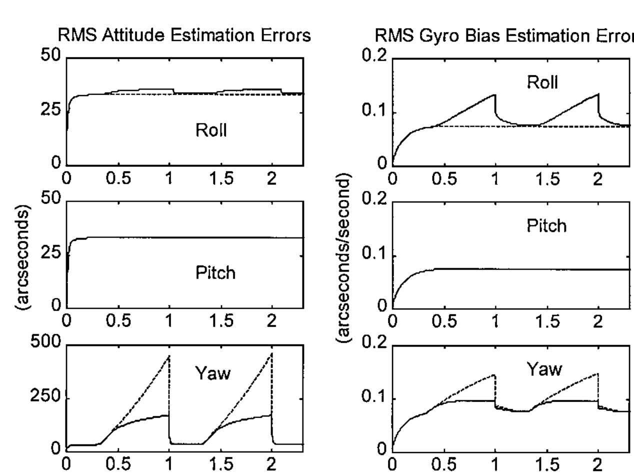


**Fig. 1 Attitude determination performance: ——, with orbital rate coupling modeled in the Kalman filter and - - -, without orbital rate coupling.**

## Conclusions

A method for reducing propagated yaw estimation errors in a satellite attitude determination system with continuous roll and pitch measurements, but with yaw measurements available only for a portion of the orbit, is presented herein. The method relies on the coupling between roll and yaw that is inherent in the kinematics of an Earth-pointing satellite. When included in a Kalman filter formulation, this coupling acts to reduce the yaw estimation uncertainty during those periods when no yaw measurements are available. The continuous roll measurements, together with the orbital rate coupling, act indirectly to improve the estimate of the yaw attitude during these periods. An analytical method for predicting the rms error of the propagated yaw estimate is also presented, and is validated with simulation results.

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# Strategy for Deployment of Multiple Satellites for Collision-Free Relative Orbital Motion


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## Introduction

ORBITAL deployment of multiple satellites by a single vehicle is becoming increasingly important, with the emergence of low-Earth-orbit (LEO) satellite constellations for mobile communications and due to other factors such as reduced cost. To achieve the constellations, several satellites are put in orbit by a single launch vehicle. They are maneuvered afterwards to put them into the proper locations. Because these operations take many days or weeks, it is essential that the satellites remain in orbit without colliding with each other for several orbits.[1] Because these deployments are carried out with a separation system of springs, inducing very small changes among the bodies, the bodies move dangerously close, in approximately the same orbits. Also, when these deployments take place either simultaneously or in quick succession within shorter time intervals, the possibility of recontact between the separated bodies is real and of concern. The multiple satellite launch by India—PSLV-C2 or Polar Satellite Launch Vehicle-Continuation Flight 2 mission—required a deployment strategy to ensure noncollision between satellites. Spagnulo and Sabathier[2] discuss inclining the satellites in the stacking configuration and selecting the incremental separation velocities to increase the relative distance between the satellites. They arrive at these values by carrying out Monte Carlo (MC) analysis by propagating the orbits of the satellites with variations in the parameters involved in the deployment for various deployment strategies. This process of carrying out MC analysis is cumbersome and computationally demanding. A strategy based on simple orbit characteristics has been devised for the deployment of these satellites so that they will not collide in the long term (less than one synodic period of the satellites involved). The usage of this strategy avoids the necessity of carrying out MC analysis (repeatedly) to assess the collision possibility for different sets of deployment parameters.

## Problem Description

Assume that from the last stage of a launch vehicle (parent body $P$), the satellite $S_1$ is separated at time $t_1$, and after a short interval of time $\Delta t$ at $t_2$ ( i.e., $\Delta t = t_2 - t_1$) another satellite $S_2$ is deployed. Because these deployments are carried out with a system of separation springs, they impart very small incremental velocities to the bodies. It is possible that satellite $S_2$ may collide with $S_1$, because they move in very close orbits. The possibility of collision exists only at the points that are common to both the orbits. If $\Delta t$ is ignored (i.e., assuming that the satellites are released at the same time), the point from which the satellites are released is one such common point. If this is the only point, then collision is possible only at this point. Obviously, after separation, to come back to this point, the satellites have to complete one revolution. There may be more than one possible collision point depending on the geometry of the orbits. The existence of such common points is only a necessary condition for a collision possibility. The sufficient condition depends on the periods of revolution. Assume that the bodies are moving under the influence of a spherical central force field only.

If $P_1$ and $P_2$ are the periods of the orbits of $S_1$ and $S_2$, then collision will occur at the common points when $P_1 = nP_2$, where $n = l$ and $l = 1, 2, 3, \ldots$, and when $n = 1/k$, where $k = 1, 2, \ldots$. If the periods of the satellites are equal ($n = 1$), they will collide at the end of one revolution or before, depending on the geometry of the orbits. Also note that if the satellites do not collide within the first few orbits, the relative distance tends to grow with time. However, the two satellites separated with small incremental velocities would come closer after one synodic period even if they do not collide in the first few orbits. In this Note, a strategy to make the period of $S_2$ equal to that of $S_1$ so that they will collide after one revolution is presented with the mathematical model. Based on this, deployment of the satellites is carried out to ensure no collision in the long term and the procedure can be extended to other values of $l$ and $k$.

## Strategy for Collision

In the separation of satellites, two parameters, namely, the incremental velocity $\Delta V$ and its direction of application (referred to hereafter as orientation) described by the angle $\theta$ between $V_P$ (instantaneous velocity vector of the parent body) and $\Delta V$, determine the orbits of the satellites. Note that the incremental velocity is shared between the parent body $\Delta V_P$ and the satellite $\Delta V_S$ in accordance with

$$\Delta V_S = +|[m_2/(m_1+m_2)]\Delta V|, \qquad \Delta V_P = |[-m_1/(m_1+m_2)]\Delta V|$$

where $m_1$ and $m_2$ are masses of the parent body and the satellite. If $\Delta V_{S_1}$ and $\theta_1$ and $\Delta V_{S_2}$ and $\theta_2$ are the separation velocities and orientations of $S_1$ and $S_2$, respectively, then $\Delta V_{S_2}$ and $\theta_2$ can be chosen so that the periods of orbits of $S_1$ and $S_2$ are equal.

## Mathematical Model

Let the satellite $S_1$ be released with $\Delta V_{S_1}$ and $\theta_1$ at $t_1$ and $V_P$ be the velocity of the parent body before separation . Let $P_1$ be the period of revolution of $S_1$ after separation. The problem is to find $\Delta V_{S_2}$ and $\theta_2$ for $S_2$ such that $P_1 = P_2$.

The period of the orbits of satellites is given by $P_i = 2\pi a_i^{3/2}/\sqrt{(\mu)}$ where $i = 1, 2$ and $a_i\{i = 1, 2\}$ are the semimajor axes and $\mu$ is gravitational constant.

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